# Statistical methods for single-cell RNA sequencing data 

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## Single-cell vs. bulk RNA-seq



Heterogeneous


Homogeneous
Sub-population


Features of single-cell RNA-seq data

- Abundance of zeros, increased variability, complex distributions


Bacher and Kendziorski, Genome Biology, 2016.

## Challenges in scRNA-seq

- Normalization
- Technical vs. biological zeros
- Clustering; Identifying sub-populations
- De-noising
- Adjusting for technical variability
- Adjusting for biological variability (oscillatory genes)
- Identifying and characterizing differences in gene-specific expression distributions (aka. identifying differential distributions)
- Pseudotime reordering
- Network reconstruction


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## - Bacher, Chu et al., Nature Methods, 2017

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Korthauer et al., Genome Biology, 2016

- Network reconstruction


# SCnorm: A quantile-regression based approach for robust normalization of single-cell RNA-seq data 

Bacher, Chu et al., Nature Methods, 2017

## Background

- Goal: correct for technical artifacts and/or gene-specific features
- Sequencing depth
- Length, GC content
- Amplification and other technical biases
- Without UMIs/spike-ins, most single-cell methods calculate global scale factors as in bulk RNA-seq
- One scale factor is calculated per sample and applied to all genes in that sample.


## Bulk: Global scale-factor normalization for sequencing depth



## Expression vs. depth varies with expression in scRNA-seq

Bulk


Single cell


We see the count-depth relationship varying with expression in many datasets

## Overview of SCnorm

- Identify gene groups based on the count-depth relationship. Within each group,
- Quantile polynomial regression is used to quantify the groupspecific relationship between expression and sequencing depth. The quantile is chosen iteratively.
- Predicted values are used to calculate group-specific scale factors for each cell.


## SCnorm

- Filter: genes having greater than $10 \%$ expression values nonzero and median nonzero expression greater than 2.
- Let $Y_{g}=\left(y_{g 1}, \ldots, y_{g J}\right)$ denote $\log$ non-zero expression for gene $g$ in cell $j ; X_{j}$ denote log sequencing depth.
- The gene-specific count-depth relationship is estimated by:

$$
Q^{0.5}\left(Y_{g, j} \mid X_{j}\right)=\beta_{g, 0}+\beta_{g, 1} X_{j}
$$

- Genes are split into $K$ groups. The group specific count-depth relationship is estimated by:

$$
Q^{\tau_{k}, d_{k}}\left(Y_{j} \mid X_{j}\right)=\beta_{0}^{\tau_{k}}+\beta_{1}^{\tau_{k}} X_{j}+\cdots+\beta_{d}^{\tau_{k}} X_{j}^{d_{k}}
$$

- Estimates of $\tau_{k}$ and $d_{k}$ minimize $\left|\hat{\eta}_{1}^{\tau_{k}}-{ }_{g}^{\text {mode }} \hat{\beta}_{\mathrm{g}, 1}\right|$; where $\hat{\eta}_{1}^{\tau_{k}}$ represents the count-depth relationship among predicted values.
- $K$ is chosen so that the absolute value of the maximum normalized slope mode is $<0.1$ within each of ten groups.


## Bulk RNA-seq



SCnorm


## Single-cell RNA-seq



SCnorm


## H1-1 ( $\sim 1$ million reads per cell)



CK SAGES 2017

## H1-4 (~4 million reads per cell)



CK SAGES 2017

## Implications for DE analysis

## $\mathrm{FC}=\mathrm{H} 1-1 / \mathrm{H} 1-4$

- H1-1: $\sim 100 \mathrm{H} 1$ cells profiles at $\sim 1$ million reads per cell
- H1-4: Same H1 cells profiled at $\sim 4$ million reads per cell
- Prior to normalization, H1-1/H1-4 should be about $1 / 4$
- Post normalization, H1-1/H1-4 should be about 1
- If over-normalization is going on, $\mathrm{H} 1-1 / \mathrm{H} 1-4$ will be greater than 1 .





## $\mathrm{FC}=\mathrm{H} 1-1 / \mathrm{H} 1-4$

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## Normalization via SCnorm



CK SAGES 2017

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scDD: A Dirichlet mixture model based approach for identifying differential distributions in scRNA-seq experiments

Korthauer et al., Genome Biology, to appear, 2016

## Gene-specific multi-modality

(A) Expression States of Gene $\mathbf{X}$ for Individual Cells Over Time


## Many genes show multi-modal expression distributions




## Opportunity to identify differences beyond traditional DE

Differential expression (DE) Differential proportions (DP)


Differential modes (DM)



Both DM and DE


## scRNA-seq DE Analysis

- Recent methods use mixture modeling to account for 'on' and 'off' components
- Shalek et al. (2014)
— SCDE (Kharchenko et al., 2014)
- MAST (Finak et al., 2015)
- When detected, each gene has a latent level of expression within a biological condition, and measurements fluctuate around that level due to biological and technical sources of variability


## scDD: Goal

- Model expression profiles while accommodating the often multimodal distributions in the detected cells
- Find genes with Differential Distributions (DD) of expression across two conditions:
— differential means
- differential proportion within modes
- differential modality (number of modes)
- combination thereof
- differential zeroes (detection rate)


## scDD: Overview

- Assume that log non-zero normalized, de-noised, expression measurements $Y_{g}=\left(y_{g 1}, \ldots, y_{g J}\right)$ for gene $g$ in $J$ cells arise from a conjugate Dirichlet Process Mixture (DPM) of normals model:

$$
\begin{aligned}
y_{j} & \sim N\left(\mu_{j}, \tau_{j}\right) \\
\mu_{j}, \tau_{j} & \sim G \\
G & \sim D P\left(\alpha, G_{0}\right) \\
G_{0} & =N G\left(m_{0}, s_{0}, a_{0} / 2,2 / b_{0}\right)
\end{aligned}
$$

- Let $K$ denote the number of components (unique values in $\left\{\mu_{\mathrm{j},} \tau_{\mathrm{j}}\right\}, j=1, . ., J$ ). Of primary interest is the posterior of $(\mu, \tau)$, which is intractable for moderate sample sizes.
- Let $Z=\left(z_{l}, \ldots, z_{J}\right)$ denote component memberships. Then $f(Y \mid Z)$ is a PPM.

$$
\begin{aligned}
f(Y \mid Z) & =\prod_{k=1}^{K} f\left(y^{(k)}\right) \\
& \propto \prod_{k=1}^{K} \frac{\Gamma\left(a_{k} / 2\right)}{\left(b_{k} / 2\right)^{a_{k} / 2}} s_{k}^{-1 / 2}
\end{aligned}
$$

## scDD: Overview (continued)

- To quantify the evidence of DD for gene $g$, obtain MAP partition estimate, $\widehat{Z}$, and evaluate $f(Y, \widehat{Z})$ under competing hypotheses:
- ignoring condition ( $\mathcal{M}_{E D}$ : equivalent distributions)
- separately within condition ( $\mathcal{M}_{D D}$ : differential distributions)
- Evaluate $\mathcal{M}_{D D}$ using a pseudo-Bayes Factor score:

$$
\text { Score }_{g}=\log \left(\frac{f\left(Y_{g}, \widehat{Z}_{g} \mid M_{D D}\right)}{f\left(Y_{g}, \widehat{Z}_{g} \mid M_{E D}\right)}\right)
$$

- Assess significance via permutation.


## scDD: Evaluation via simulation studies

- 8000 ED genes:
- 4000 from single Negative Binomial component
- 4000 from two component mixture of Negative Binomial

■ 2000 DD genes:

- 500 DE genes
- 500 DP genes ( $0.33 / 0.66$ proportion difference)
- 500 DM genes ( 0.50 belong to second mode)
- 500 DB genes (mean in second condition is average of means in the first)
- Sample sizes varied $\in\{50,75,100\}$
- Component distances $\Delta_{\mu}$ for multimodal conditions varied $\in\{2,3,4,5,6\}$ SDs
■ Means, variances, and detection rates sampled empirically

Evaluate: Power to identify DD genes
Rate at which DD genes are correctly classified Rate at which correct \# components are identified

## scDD: Power to detect DD genes within each category

| Sample Size | True Gene Category |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Method | DE | DP | DM | DB | Overall (FDR) |
|  | scDD | 0.893 | $\mathbf{0 . 4 1 8}$ | $\mathbf{0 . 8 9 8}$ | $\mathbf{0 . 5 7 2}$ | $\mathbf{0 . 6 9 5}(0.030)$ |
|  | SCDE | 0.872 | 0.026 | 0.816 | 0.260 | $0.494(0.004)$ |
|  | MAST | $\mathbf{0 . 9 0 8}$ | 0.400 | 0.871 | 0.019 | $0.550(0.026)$ |
| 100 | scDD | 0.951 | 0.590 | $\mathbf{0 . 9 6 0}$ | $\mathbf{0 . 6 6 8}$ | $\mathbf{0 . 7 9 2 ( 0 . 0 3 1 )}$ |
|  | SCDE | 0.948 | 0.070 | 0.903 | 0.387 | $0.577(0.003)$ |
|  | MAST | $\mathbf{0 . 9 5 6}$ | $\mathbf{0 . 6 3 2}$ | 0.942 | 0.036 | $0.642(0.022)$ |
|  | scDD | 0.972 | 0.717 | $\mathbf{0 . 9 8 2}$ | $\mathbf{0 . 7 2 7}$ | $\mathbf{0 . 8 5 0}(0.033)$ |
|  | SCDE | 0.975 | 0.125 | 0.946 | 0.478 | $0.631(0.003)$ |
|  | MAST | $\mathbf{0 . 9 7 7}$ | $\mathbf{0 . 7 5 2}$ | 0.970 | 0.045 | $0.686(0.022)$ |
|  | sCDD | $\mathbf{1 . 0 0 0}$ | 0.985 | $\mathbf{1 . 0 0}$ | $\mathbf{0 . 9 0 3}$ | $\mathbf{0 . 9 7 2 ( 0 . 0 3 4 )}$ |
|  | SCDE | $\mathbf{1 . 0 0 0}$ | 0.858 | 0.998 | 0.785 | $0.910(0.004)$ |
|  | MAST | $\mathbf{1 . 0 0 0}$ | $\mathbf{0 . 9 9 2}$ | $\mathbf{1 . 0 0}$ | 0.174 | $0.792(0.021)$ |

## Comparison of hESCs



## Number of DD genes identified in each cell type comparison

|  | scDD |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Comparison | DE | DP | DM | DB | DZ | Total | SCDE | MAST |
| H1 vs NPC | 1342 | 429 | 739 | 406 | 1590 | 4506 | 2938 | 5729 |
| H1 vs DEC | 1408 | 404 | 939 | 345 | 880 | 3976 | 1581 | 3523 |
| NPC vs DEC | 1245 | 449 | 700 | 298 | 2052 | 4744 | 1881 | 5383 |
| H1 vs H9 | 194 | 84 | 55 | 32 | 145 | 510 | 102 | 1091 |
| scDD only: | $2 \%$ | $21 \%$ | $38 \%$ | $24 \%$ | $15 \%$ |  |  |  |

## Genes identified in H1 vs. NPC comparison




